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The fluid relation between reading strategies and mathematics learning: A perspective of the Island Ridge Curve

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ABSTRACT

Mathematics learning is not only determined by mathematical contents, but also by reading strategies that students apply to comprehend the mathematical tasks. Although studies in mathematics education have shown that reading strategies are positively related to mathematics learning, the relation may not be linear as widely believed. Recent research in language assessment has found that the association between reading strategies and reading performance fluctuates as language proficiency increases, a pattern metaphorically called the Island Ridge Curve (IRC). Inspired by the IRC, we hypothesize that the relation between reading strategies and mathematics learning also fluctuates in the pattern of the IRC as students' reading proficiency increases. To verify this assumption, we used a public dataset of 529,091 15-year-old students (M = 15.79, SD = 0.29) from 77 countries/territories. Results of multilevel mixture regression analysis indicated that reading strategies (in our case, understanding and memorizing strategies) are related to mathematics learning through reading proficiency and this relation fluctuates like the IRC. Implications for applying IRC to reading strategies instruction in mathematics education are discussed. (170 words).

1. Introduction

Mathematical literacy refers to 'an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts' (OECD, 2019, p. 75). Mathematical literacy plays an essential role in children's academic development (Kikas et al., 2020; Morgan et al., 2014; Ritchie and Bates, 2013). However, adolescents in formal education are often found floundering in mathematics learning (Pape, 2004; Wu et al., 2021). An increasing volume of research has shown this challenge not only comes from students' mathematical knowledge and skills, but also from their reading proficiency (Hagena et al., 2017; Morgan et al., 2014; Planas and Schütte, 2018) and more specifically, from their use of reading strategies such as comprehending and memorizing strategies that students utilize to comprehend mathematical materials (Björn et al., 2016; Capraro et al., 2012; Grimm, 2008). The importance of reading proficiency and reading strategies to mathematics learning becomes more salient when mathematics learning in higher grades evolves to be more textbook-based (Pape, 2004; Wu et al., 2021).

Existing studies on reading strategies are usually underlined by a monotonous view, conceiving that the more strategies students use, the better outcome in their achievement (Ng et al., 2021; Usta and Yılmaz, 2020). Most recent findings from language research, however, cast doubt over this belief. An increasing number of studies accounting for the moderation of language proficiency on this strategy-language relation show that this fluctuates as language proficiency increases (Hong-Nam and Leavell, 2006; Hong-Nam and Page, 2014; Purpura, 1999), a phenomenon called the Island Ridge Curve (IRC; Cai, 2020, 2022; Cai and Kunnan, 2019, 2020; Cai and Cheung, 2021; Cai and Chen, 2022).

Moving further, we posit that the association between reading strategies and mathematics achievement also fluctuates. The current study aimed to examine the universalness of the IRC for interpreting the relations among students' perceived usefulness of reading strategy use (i.e., understanding and memorizing strategies), reading proficiency, and mathematics achievement in PISA 2018 data.

2. Literature

2.1. Mathematics learning and reading

Mathematics learning not only involves the knowledge and skills in

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mathematics but also the language carrying these mathematical skills and the reading strategies for comprehending such information (National Council of Teachers of Mathematics (National Council of Teachers of Mathematics (NCTM), 2000). Mathematics problems distinguish between bare problems and word problems. Bare problems are presented using mathematical symbols or expressions without contexts provided (e.g., 3 + 6 =?), while word problems depict the problem scenarios primarily using linguistic forms (Verschaffel et al., 2020). Solving bare problems relies more on students' ability to understand mathematical background knowledge and therefore is less dependent on reading proficiency (Kan et al., 2019), whereas solving word problems is more demanding on reading proficiency which enables the students to convert the linguistic components into mathematical expressions and equations (Kan et al., 2019).

Correspondingly, the literature has distinguished two types of difficulties in mathematics learning: difficulty in mathematics (e.g., digit addition) and difficulty in reading (Dirks et al., 2008; Geary, 2004; Rubinsten and Henik, 2009). Evidence shows that students with both difficulties achieve lower in mathematics learning than those only with mathematics difficulties (Fuchs and Fuchs, 2002; Vukovic et al., 2010).

A major aspect of the reading difficulty relates to the high demand that mathematical tasks bear on students to read and understand the tasks (Capraro et al., 2012; Fuentes, 1998; Pape, 2004; Sanz et al., 2020; Usta and Yılmaz, 2020). Mathematical tasks are usually presented with texts describing the problem which requires a sufficient level of reading proficiency. Mathematical knowledge, such as mathematical principles, concepts, and proofs, is usually conveyed and acquired through reading materials (Mejia-Ramos et al., 2012; Österholm and Bergqvist, 2013; Shepherd and Van De Sande, 2014). These materials are characteristic of multi-semiotic and highly technical language usually written in a compact style where meanings, concepts, and relations of sentences are implicitly embedded (O'Halloran, 1998; Wilkinson, 2019). While for aesthetic purposes this ambiguity is tolerable or even acceptable, for mathematics learning the ambiguity must be resolved and the precise meaning must be determined (Fuentes, 1998).

To address these issues, scholars in mathematics education drew on reading theories (Kintsch, 1998; Kintsch and Mangalath, 2011) and proposed a two-phase model for comprehension during mathematics problem solving: the representation phase and the solution phase (Hegarty et al., 1992; Krawec, 2014; Mayer, 1992). During the first phase, linguistic components such as words, clauses, and sentences are deciphered to obtain the literal meaning. Connections between individual propositions are identified and a mental representation that reflects the state of affairs described by the text is constructed (Kintsch et al., 2005; Pape, 2004). During the second phase, solutions (e.g., choosing appropriate equations) are generated and planned based on the mental representation, and the algorithmic operation is performed to produce the answer. In this way, reading comprehension lays a foundation for prior knowledge retrieval and the proper formation of solutions during mathematics learning (Fuchs et al., 2008; Fuentes, 1998; Hadianto et al., 2021).

This relation between reading comprehension and mathematics learning has found support from empirical studies (e.g., Björn et al., 2016; Erbeli et al., 2021; Grimm, 2008). Grimm (2008) tracked the association between reading comprehension and mathematics performance with a cohort of students from Grade 3 to Grade 8. The results showed students' improvement showed that all reading subskills significantly predicted equation formation. The researchers conceptualized a two-stage model to explain the mechanism of reading strategies: the problem translation stage and the problem integration stage. The first is where lower-order strategies function to construct a basic mental representation, and the second is where higher-order strategies help build an integrated and coherent representation of the problem. This two-stage model is indeed similar to the two-phase model of mathematics problem solving (representation-solution) (Mayer, 1992; Pape, 2004).

Regardless, inconsistent findings emerged. Ng (2006) compared the effect of reading strategies (rehearsal, elaboration, and organizing strategies) on the mathematical performance of gifted students from Grade 6 to Grade 8. Results of the correlational analysis showed that these reading strategies worked with students in senior middle school but not with those in junior high schools. The researchers interpreted the significant effect in senior middle school as the result of the higher demand for reading strategies by higher grade mathematical tasks. Alternatively, the higher reading proficiency of students in senior middle school might also have contributed to the significance.

Hagena et al. (2017) experimented with the effect of instruction on reading strategies on mathematics competence with 380 13-year-old students. Students were put into three training conditions: integrated strategy (Experiment Group 1), separated reading strategy (Experient Group 2), and no strategy (Control Group). In the end, the researchers identified a significant improvement in mathematics achievement in all groups but the between-group difference in the improvement were not significant. The researchers attributed the non-significance to method issues (e.g., test quality, intervention duration, etc.). As the study did not provide information regarding the language or reading proficiency of the Control Group, it was unclear whether reading proficiency had played its role.

The mixed findings generated from the literature prompts us to consider other important factors such as reading proficiency that might have blurred our understanding of the relation between reading strategies and mathematics learning. The next section reviews an emerging theory of the Island Ridge Curve (IRC) that may help us to look into this complex issue.

2.3. The Island Ridge Curve (IRC)

The Island Ridge Curve (IRC) is an emerging theory explaining the mechanism in which language proficiency moderates the relation between cognition factors and language performance. It was first introduced by Cai (2020) and Cai and Kunnan (2019, 2020) in reading assessment. The IRC had its origin from the short-circuit hypothesis (Clarke, 1980a, 1980b) in language research. The short-circuit theory contends that the knowledge and skills a learner acquired during first language learning (e.g., prior knowledge, strategies) can be transferred to second language learning. However, this transfer presupposes that the learners' second language proficiency should pass a certain threshold; otherwise, the activation of these skills would be short-circuited.

When studying the relation of background knowledge to second language performance, Clapham (1996) separated students into three groups using two subjective cutoff points of students' second language test scores (60 % and 80 % out of the total score). She compared the effect of background knowledge on reading performance across the three groups and found the highest effect with the middle group. Drawing on this finding, Clapham updated Clarke's one-threshold hypothesis to a two-threshold hypothesis.

To overcome the limitations of using subjectively determined cutoff scores, Cai (2020) developed an advanced statistical model called multi-layered moderation analysis which allows the projection of the varying effect of one predictor (i.e., reading strategies) on the dependent variable (e.g., medical English reading performance) onto the whole continuum of the moderator (i.e., general English language proficiency). Taking advantage of this analytical technique, Cai and Kunnan (2020) examined the interaction between reading strategies and language proficiency and found the relation between reading strategies and medical reading performance varied as language proficiency continuously increased. Specifically, the variation followed a 'down-up-down' pattern, thereby the metaphorical label of the Island Ridge Curve (IRC). The IRC revealed three language thresholds where the trajectory of strategy effect made turns: −1.29, −0.0.71, and 1.29 standard units. Divided by these thresholds, students were grouped into divers (with whom strategy effect was negative and deteriorated as language proficiency increased), resurfacers (with whom strategy effect was negative but gradually leveled off as language proficiency increased), uphillers (with whom strategy effect became positive and gradually increased), and downhiller (within whom strategy effect remained positive but gradually stepped down). Please refer to Fig. 1 for the original picture illustrating the IRC mechanism.

In the original IRC study, Cai and Kunnan (2020) explained that the effect of reading strategies on medical English reading reaches its peak when language proficiency falls within the medium level. This is because in this central area language proficiency is good enough to release the beneficial potential of the cognitive factors that are constrained in the low-proficiency area. However, when language proficiency increases beyond the higher threshold, internalized language proficiency itself can automatically process text information such that more activation of strategies becomes redundant. Since the debut of IRC, empirical evidence supporting the IRC has been accumulating. Examples include studies examining learner strategy use (García-Pérez et al., 2021; Gui et al., 2021), motivation regulation strategies in writing (Cai and Chen 2022), and medical knowledge in reading (Cai and Kunnan, 2019).

Most recently, Cai (2022) and Cai and Chen (2022) conceive two theorems to explain the performance of the IRC: *bipolarity* and *golden centrality*. Bipolarity means any motion at a particular moment has a bifurcating moving tendency (i.e., upward or downward). Golden centrality corresponds to the 'golden mean' posited by Aristotelian philosophers (Bartlett and Collins, 2011). These philosophers argue that human excellence (e.g., virtuous disposition) lies in a middle point between two extremes. Arguably, language proficiency and reading strategies could be on this list of human nature.

Although the IRC was originally developed by examining the static change (i.e., inter-person variation) during learning, the IRC can also provide new thoughts for interpreting the trajectory of learning development (i.e., intra-person variation). Learning does not take place automatically and is transformed from the continuous drive of multiple factors such as cognition (e.g., prior knowledge, metacognitive and cognitive strategies, executive functions), motivation (e.g., enjoyment, goals, interest), cognition-based affection (e.g., growth mindset, self-concept, self-efficacy, motivation regulation, emotion regulation) (Moreno, 2010; Plass and Kalyuga, 2019), among other individual and contextual factors. During this process, the activation of cognition and cognition-related factors is energy-consuming and leads to increased cognitive load (Boekaerts, 2017; Feldon et al., 2019; Seufert, 2018; Wirth et al., 2009; Wirth et al., 2020; Xu et al., 2021). The activation may bring about gains when the benefit outweighs the cost; otherwise, the activation may render a loss. The theorem of bipolarity acknowledges the bifurcating moving orientations of a factor at a particular temporal point on the trajectory of learning development.

Moreover, much like the lifecycle of a biological entity, the magnitude of the contributions by these factors experiences stages from infancy, adolescence, adulthood, and late adulthood. It is weaker and softer during the early stages, grows up strong and prosperous during its 'golden adulthood', and fades out during late adulthood. Besides, different factors vary in terms of the timing they reach their 'golden adulthood'. To facilitate optimal learning, these factors adapt their contributions collaboratively, that is, they automatically tune down or tune up their strengths of activation.

This adaptability can find empirical support from a large-scale

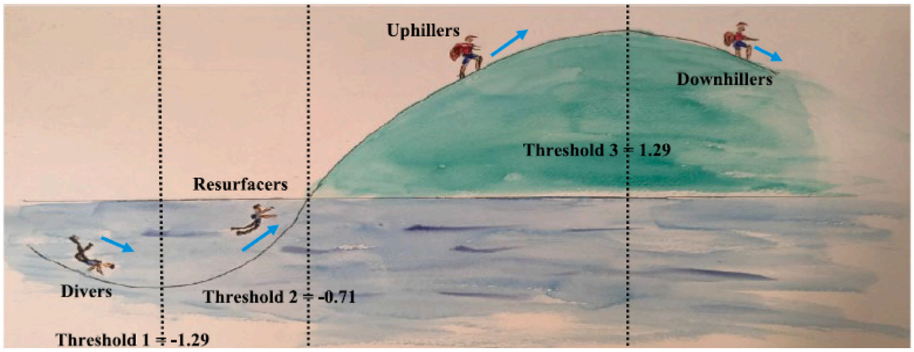

Fig. 1. A metaphoric illustration of the island ridge curve (IRC) (from Cai and Kunnan, 2020, p296).

longitudinal study conducted in Hong Kong (Cai, 2019). The study involved 2473 7th Graders from 16 junior middle schools in Hong Kong. Student data on strategy use and English achievement were collected three waves within three years at the interval of one year. Results of latent profile analysis showed that students can be classified into low-, medium-, and high-frequency strategy users across each wave. Results of latent transition analysis showed that the class memberships transferred across waves in such a pattern that medium-frequency strategy users remained stable but both low-frequency and high-frequency strategy users showed a tendency to transfer to the middle-frequency users. Moreover, the results of the ANOVA test indicated the medium-frequency users obtained the highest score in English achievement across each wave.

An important note about the IRC is that, existing evidence regarding the pattern of the IRC suggests an italicized s shape illustrating variation of the effects of cognitive factors due to the moderation of language proficiency. However, this s shape should not be regarded as determinative. When the moderated factors are of different nature (e.g., other cognitive factors such as critical thinking, systems thinking, design thinking, and prior knowledge, or cognition-based motivation factors such as motivation regulation, self-concept, and growth mindset), the s shape might rotate its angle to a certain degree due to the different natures of the factors being moderated. To determine whether the IRC is at work, one should both consider whether a moderated effect fluctuates and whether the largest effect resides somewhere in the middle of the continuum of language proficiency.

Given the promising future of the emerging IRC, we step further and make an ambitious position that learning in all domains mediated by language (e.g., mathematics, science, economics, etc.) are constrained by language proficiency, and this interference could follow the rules of the IRC. As a language-mediated human development activity, mathematical learning should also depend on learners' language proficiency (i.e., reading proficiency) that students rely on to study the subject and on the reading strategies that students use to comprehend mathematical materials.

2.4. Covariates effects on reading and mathematics achievement

Numerous studies have reported that students' learning achievements are influenced by personal factors such as student gender, socioeconomic status (SES), and home language. The literature generally shows that gender affects learning by favoring girls in reading (Chiu and McBride-Chang, 2006; Logan and Johnston, 2009; Nalipay et al., 2020) and favoring boys in mathematics (Baiduri et al., 2020; Liu and Wilson, 2009), that SES is positively related to learning in general (Bernardo et al., 2021; Marks and O'Connell, 2021), and that students using the test language at home outperform their peers using another language in first language reading (Babayiğit and Shapiro, 2020; Raudszus et al., 2019) but not necessarily in mathematics (Gilleece et al., 2010; Prinsloo and Harvey, 2020). To control for possible confounding effects, student gender, SES, and home language were included as covariates in our analyses.

The purpose of the current study was to explore the relations among reading strategies, reading proficiency, and mathematics achievement through the lens of the IRC, specifically, to explore whether the relation of reading strategies to mathematics achievement fluctuates with students' increase in reading proficiency.

The current study was led by two questions:

1. *To what extent is reading proficiency associated with mathematics achievement across students of different reading proficiency?*
2. *Does the association between reading strategies and mathematics achievement fluctuate across students of different reading proficiency? If the answer is yes, in what pattern?*

3. Method

3.1. Data

The current study used OECD PISA 2018 data from students from 77 countries/territories. The data are publicly available (https://www.oecd.org/pisa/data/2018database/) and no ethics clearance approval was involved for the authors' data collection. Specifically, we used student response data on reading strategies (i.e., understanding and memorizing strategies), reading and mathematics test scores. After dropping cases with missing values on these key variables, we retained a total of 529,091 students (51 % girls, mean age = 15.79, SD = 0.29). Among them, 83 % spoke the test language at home.

Socio-economic status (SES) in PISA 2018 was represented by ESCS (an index of economic, social, and cultural status) that contained information of students' family background (e.g., their parents' education and occupation, home possessions, and cultural resources) (OECD, 2019). The mean of ESCS for our data was −0.20 (SD = 1.07).

Table 1 shows detailed student information by groups including student age, SES, and distributions of gender and home language.

3.2. Measures

3.2.1. Mathematical literacy

PISA 2018 does not provide actual scores but 10 plausible values for mathematics literacy, each representing a random value drawn from the posterior distribution of the actual score of a student. For our study, the first plausible value was used to represent mathematical achievement as previous studies have found that results from different plausible values are identical (Spiezia, 2011). The mean of the plausible value was 476.03 (SD = 103.35).

3.2.2. Reading literacy

As with mathematical literacy, PISA 2018 provided 10 plausible values for reading literacy and we also used the first plausible value to

Table 1
Student information across four reading groups.

Group	Struggling readers (n = 55,367)	Low-proficiency (n = 78,067)	Medium-proficiency (n = 340,428)	High-proficiency (n = 55,229)
Gender	Girls =35 %	Girls =46 %	Girls = 54 %	Girls = 58 %
SES	−0.93 (SD = 1.19)	−0.72(SD = 1.10)	−0.13(SD = 1.01)	0.49(SD = 0.82)
Age	15.77(SD = 0.29)	15.78(SD = 0.29)	15.79(SD = 0.29)	15.81(SD = 0.29)
Home language	Test language =71 %	Test language =78 %	Test language =85 %	Test language =90 %

represent reading proficiency. The mean of the plausible value was 466.95 (SD = 104.66).

3.2.3. *Reading strategies*

Reading strategies were measured with a six-point scale (*1 = not useful at all, 6 = very useful*) that asked students to respond to the question "*How do you rate the usefulness of the following strategies for understanding and memorizing the text?*". The question was followed by six statements, each on one type of strategy. For detailed information regarding the statements please refer to Table 2. The overall mean was 3.72 (SD =1.63) and the internal consistency of the six items was α = 0.74.

3.3. *Data analysis*

We first put students into four groups using three reference scores discovered in the original IRC study (Cai and Kunnan, 2020), namely, −1.29, −0.71, and 1.29 standard units of reading proficiency, corresponding to the raw PISA scores of 332, 393, and 602, respectively. These four groups were labelled as struggling readers (n = 55,367, 11 % of the total sample), low-proficiency readers (n = 78,067, 15 %), medium-proficiency readers (n = 340,428, 64 %), and high-proficiency readers (n = 55,229, 10 %), respectively. Next, we conducted multilevel mixture modeling with the school as the cluster variable to explore the relations among reading strategies, reading proficiency, and mathematics achievement at the student level and the variation of this relation across groups. To control for possible confounding effects, gender, student SES and their home language were also included in our model.

Three points deserve notice for this multilevel approach. First, we only focused on student-level relations and used the multilevel structure only to filter out variances at or beyond the school level (see Cai et al., 2019). Second, we used the Rasch-calibrated single score (UNDREM) for reading strategies stored in the PISA 2018 dataset for its computational simplicity and its ability in filtering out measurement errors (OECD, 2020). Third, to reduce estimation bias due to sampling, we used the senate weight suggested by OECD (2009) in our analysis.

This multilevel mixture modeling was run on M*plus* 8.5 (Muthén and Muthén, 1998-2020) with reading proficiency groups as the known class with the estimator of full information maximum likelihood. Before conducting the multilevel mixture modeling, we tested the model-data fit of a traditional multilevel model combing all students. The quality of the model was evaluated based on multiple criteria: RMSEA (Root Mean Square Error of Approximation) and SRMR (Standardized Root Mean Square Residual) no larger than 0.05, and TLI (Tucker–Lewis index) and CFI (Comparative Fit Index) no smaller than 0.95 (Mueller and Hancock, 2010).

4. Results

4.1. *Preliminary analysis*

Table 3 shows the correlations among the key variables across groups. As shown, mathematics achievement was positively associated with reading across all groups: rs = 0.37, 0.22, 0.63, and 0.42 (all with $p < .01$) for the struggling-, low-proficiency, medium-proficiency, and high-proficiency readers, respectively, suggesting the IRC pattern. A similar pattern was suggested by the association between mathematics and reading strategies (rs = 0.02, 0.03, 0.15, and 0.04, respectively, all with $p < .01$), as well as by the association between reading strategies and reading (rs = 0.05, 0.06, 0.22, and 0.06, respectively, all with p < .01).

4.2. *Multilevel mixture modeling*

The single-group multilevel model produced an excellent fit at the student level: TLI = 1.000, CFI =1.000, RMSEA =0.000, and SRMR = 0.000 (within). Drawing on this structure, we conducted a multilevel mixture path analysis with the school as the cluster variable and reading proficiency groups as the known classes. At the student level, mathematics was regressed on reading strategies through reading proficiency. Besides, all key variables were regressed on the covariates (i.e., gender, SES, and home language). The results are shown in Fig. 2.

The effects of reading strategies on reading across the four groups were βs = 0.03, 0.12, 0.34, and 0.05 (all with $p < .001$), respectively. Results of the Wald Test show the difference between the medium-proficiency readers and the other three groups were all statistically significant. From struggling to high-reading proficiency, the difference test statistics were: X^2/df = 2579.276/1, 6905.630/1, and 2318.306/1, all with $p < .001$. These results suggested the IRC pattern for which reading proficiency moderated the relation between reading strategy and reading.

The relation of reading to mathematics across the four groups were βs = 0.51, 0.53, 0.58, and 0.50 (all with $p < .001$), respectively. Results of Wald Test suggested that the effects between the struggling readers

Table 2
Descriptive statistics of reading strategies by groups.

Code	Content	Struggling		Low		Medium		High	
		M	SD	M	SD	M	SD	M	SD
ST164Q01	I concentrate on the parts of the text that are easy to understand.	3.22	1.82	3.66	1.70	3.62	1.55	3.21	1.46
ST164Q02	I quickly read through the text twice.	3.12	1.67	3.35	1.65	3.21	1.57	3.06	1.48
ST164Q03	After reading the text, I discuss its content with other people.	3.19	1.72	3.46	1.72	3.75	1.64	4.20	1.50
ST164Q04	I underline important parts of the text.	3.58	1.78	4.04	1.73	4.45	1.57	4.58	1.43
ST164Q05	I summarise the text in my own words.	3.56	1.77	3.98	1.70	4.43	1.52	4.66	1.38
ST164Q06	I read the text aloud to another person.	3.26	1.86	3.27	1.82	3.17	1.71	3.14	1.58
Overall		3.32	1.39	3.63	1.25	3.77	0.99	3.81	0.81
Math		332.11	64.03	387.20	60.15	489.38	75.13	612.75	61.90
Reading		289.75	33.86	364.50	17.35	490.16	57.24	646.29	37.05

Overall Cronbach's Alpha = 0.74.

Table 3
Correlations across reading proficiency groups.

Variable	Struggling readers					Low-proficiency readers					Medium-proficiency readers					High-proficiency readers				
	V2	V3	V4	V5	V6	V2	V3	V4	V5	V6	V2	V3	V4	V5	V6	V2	V3	V4	V5	V6
V1. Math	0.37*	0.02*	0.19*	0.14*	−0.03*	0.22*	0.03*	0.21*	0.22*	−0.02*	0.63*	0.15*	0.28*	0.17*	−0.03*	0.42*	0.04*	0.09*	0.23*	−0.01
V2. Reading		0.05*	0.03*	−0.11*	−0.03		0.06*	0.07*	−0.04*	−0.01*		0.22*	0.26*	−0.04*	−0.03*		0.06*	0.13*	−0.02*	−0.004
V3. Strategy			−0.02*	−0.05*	0.00			−0.01*	−0.08*	0.01			0.06*	−0.10*	−0.01*			0.03*	−0.12*	0.00
V4. SES				0.13*	−0.01				0.10*	0.01				0.03*	−0.01*				−0.01	0.001
V5. Gender					0.01*					0.003					−0.002					−0.003

Note: V6 = home language.
* $p < .01$.

and the low-proficiency readers were not statistically significant (Wald Test: X^2 /df = 0.348/1, p = .555); but the effect with the medium-proficiency readers was significantly different from the low-proficiency readers (Wald Test: X^2 /df = 17.578/1, $p < .001$) and from the high-proficiency readers (Wald Test: X^2 /df = 166.985/1, $p < .001$). The relation between reading proficiency and mathematics achievement across the four proficiency groups seemed to fluctuate in the IRC pattern.

The indirect effects of reading strategies on mathematics through reading across the four groups were 0.02, 0.06, 0.20, and 0.03, respectively, all with $p < .001$. Results of the difference test indicated a significant difference between the medium-proficiency readers and each of the other groups: X^2 /df = 19.21/1 (between struggling readers and low-proficiency readers), 6292.05/1 (between low-proficiency and medium-proficiency readers), and 2788.29/1 (between medium-proficiency and high-proficiency readers), all with p < .001. The largest effect of mediated effect of reading strategies on mathematics was with the medium-proficiency readers.

Note that the multilevel mixture modeling showed a small but positive effect of reading strategies on reading with struggling readers and the results were inconsistent with the negative effect found with the 'divers' in the original IRC. We assumed the difference should have come from the modeling of reading proficiency as a nominal variable in the current study as against the use of language proficiency as a continuous variable in the original IRC. Therefore, we went back to the PISA data with students at the extreme lower end of reading proficiency and discovered a negative association between reading strategies and mathematics achievement with students scored below 189 ($r = -0.086$, $p < .05$) and students scored between 189 and 262 ($r = -0.023$, p < .05). Note that this supplementary analysis is better regarded as a zooming-in exploration for more detailed information beyond the collective performance of the struggling readers in the current study, rather than a reframing of the rationale underlying the current study that is against the IRC.

Regarding covariate effects, across all groups gender was positively associated with mathematics in favor of boys (βs = 0.15, 16, 0.19, and 0.22, all with $p < .001$) and negatively associated with reading in favor of girls with trivial or non-significant effect sizes (βs = −0.08 (p < .001), −0.01($p < .01$), −0.04 (p < .001), and − 0.01 ($p > .05$), respectively). The relation of home language to mathematics achievement was not significant except for the trivial negative effect with the low- and the medium-group (βs = −0.02, $p < .05$) and the relation of home language to reading were all negative but with trivial effect sizes (βs = −0.02, −0.08, −0.03, and − 0.01, all with p < .05).

5. Discussion

1. To what extent is reading proficiency associated with mathematics achievement across students of different reading proficiency?

Our study showed that reading proficiency was positively related to mathematics achievement across various reading proficiency groups. The strong association between reading proficiency and mathematics achievement can also be explained by the common nature of reading and mathematics as human knowledge domains. In PISA 2018, both reading and mathematics are defined as one type of human literacy (OECD, 2019), both share features of cognitive processes such as assessing, interpreting, evaluating, and so forth (OECD, 2019).

The significant relation substantiates the two-phase model in which reading proficiency facilitates mathematical problem-solving: the representation phase and the situation phase (Kintsch, 1998; Kintsch and Mangalath, 2011). During the first phase, students activate their linguistic resources which not only enable them to comprehend the text describing the context but also that carrying subject-related content (Kintsch et al., 2005; Pape, 2004). During the second phase, students come up with mathematical solutions and perform mathematical

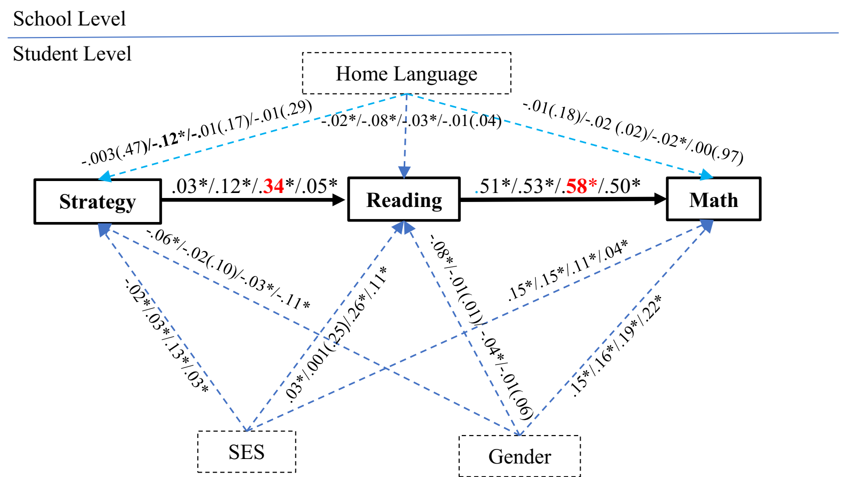

Fig. 2. Results of multilevel mixture modeling with standardized estimates.
* p < .001; *p* values larger than *p* = .001 are presented in the brackets; Gender: 1 = females, 2 = males; Home language: 1 = test language, 2 = other languages;

operations. During these two phases, students not only activate their linguistic knowledge, but also cognitive resources such as reading strategies, general world knowledge, and subject knowledge in mathematics (Fuentes, 1998; Hadianto et al., 2021).

Most interestingly, we found that the relation between reading proficiency and mathematics achievement varied across students of different reading proficiency. Specifically, the effect projected out with students in the medium-proficiency group, forming an 'up-down' pattern across all four groups. Recall that the original IRC (Cai and Kunnan, 2020) disclosed a 'down-up-down' pattern illustrating the motion of strategy-reading relation with the increase of language proficiency. In the study, the first 'down' motion was observed with the 'divers' (corresponding to struggling readers in our study), the 'up' motion with the 'resurfacers' (lower-proficiency readers) and uphillers (medium-proficiency readers), and the second 'down' motion with the downhillers (high-proficiency readers). A comparison between the current observation with the original IRC shows the similarity except for two differences. First, the effects with the divers and resurfacers were negative in the original IRC but positive in our study. The second difference is the absence of the 'down' motion with the struggling readers in our current study. The most possible explanation for these two differences should relate to the nature of the two attributes under study. Reading proficiency as a 'trait' variable is more stable once acquired, whereas strategy is more of a 'state' that is prone to change when confronted with a challenge (Phakiti, 2008).

The larger effect of reading proficiency with the medium-proficiency readers followed by a decrease with the high-proficiency readers provides supporting evidence for the theorem of 'golden centrality' in the IRC literature (Cai and Chen, 2022). In line with Aristotelean philosophers (e.g., Bartlett and Collins, 2011), language proficiency as a type of human nature functions most efficiently if optimally applied during human learning activities.

The decreased effect with the higher-proficiency readers also corroborates the rule of 'bipolarity', which contends that language proficiency does not need to fall into the 'the more, the better' rule when exerting its influence.

A further explanation of this effect drop with the high-proficiency readers could relate to the idea of 'effect saturation'. The coined term of effect saturation means that the potential magnitude of an effect is definite even when the facilitation process is perfectly conducted. Take mathematical problem-solving as an example. A mathematics problem is a reading problem as it involves reading to comprehend the text describing the task. Nevertheless, a mathematical problem is 'mathematics' and would after all rely on the successful activation of mathematics schemata. After reading proficiency has released its highest potential, more activation of reading schemata would produce more cognitive load (Shehab and Nussbaum, 2015; Wirth et al., 2020) which would bring about less benefit but could be saved for the helpful activation of mathematics schemata.

The relatively smaller effect of reading proficiency with the struggling readers and lower-proficiency readers suggests the existence of a certain linguistic threshold (Clarke, 1980a, 1980b). Erbeli et al. (2021) found that reading proficiency made little contribution with lower-proficiency students but the effect became salient with average-and high-level learners. Although they did not ascribe the difference to reading proficiency, there is a strong signal that some language threshold(s) might be in action. In our case, this threshold could be somewhere near the standard units of −0.71 (or 365, slightly below the grand mean of 400).

In all, although the current study did not replicate the exact pattern of the original IRC, our findings are in general consistent with the IRC: the fluctuation of the effect of reading on mathematics and the largest effect retained with the medium-proficiency achievers.

2. Does the association between reading strategies and mathematics achievement fluctuate across students of different reading proficiency? If the answer is yes, in what pattern?

Our results showed that reading strategies were positively related to mathematics achievement through reading proficiency, and this relation fluctuated across different reading proficiency groups, with the strongest relation for the medium-proficiency readers and small or negligible relations among other groups. First, this overall fluctuation mainly came from the fluctuation of the relation between reading strategies and reading proficiency. This fluctuation pattern is in general consistent with the original IRC (Cai and Kunnan, 2020) as well as with what was found in other studies set in second language reading (Hong-Nam and Leavell, 2006; Hong-Nam and Page, 2014).

In our case, the struggling readers and low-proficiency readers corresponded to the divers and resurfacers in the original IRC. Note that in the original IRC study, a negative relation of reading strategies to reading performance was observed with the divers (corresponding to struggling readers) and the resurfacers (low-proficiency readers), but in the current study, the relation was both positive and trivial. This slight difference should mostly be due to the granularity of statistical analysis. In the original IRC, a parametric method was used for group detection, whereas in the current study a non-parametric way was used to group the participants. Results produced in the current study, hence, can be regarded as compromised representation of results from more granular groups using the parametric group detection method.

A possible interpretation for the small difference should relate to how reading strategies were measured. In the original IRC study, reading strategies contained multiple dimensions such as planning, monitoring, evaluating, apart from the understanding and memorizing measured in the current study. The inclusion of various reading strategies in the original IRC study may incur more instability.

Another possible interpretation for the difference should concern the context of strategy use. In the original IRC, reading strategies were used in the second language context and related to medical English learning, whereas the current study was set the first language context-oriented toward mathematics learning. Strategy use may be more stable in the native language context than in second language settings (Clarke, 1980a, 1980b).

An alternative interpretation relates to technical issues. The current study was based on the comparison of nominal groups which only allowed us to provide a general picture of the variation, whereas in the original IRC study the MLMA technique allowed a continuous projection of the effect (Cai and Kunnan, 2019, 2020). The negative relation between reading strategies and reading proficiency might have been absorbed during the comparison between nominal groups (e.g., Hong-Nam and Leavell, 2006; Hong-Nam and Page, 2014). The negative relation between reading strategy use and reading proficiency discovered through our supplementary analysis confirmed this explanation (please see Supplementary Table 1).

Concerning the overall relation between reading strategies and mathematics achievement, the positive but trivial relation between reading strategies and mathematics achievement with the struggling and low-proficiency readers is consistent with what was found in Ng (2006) and Hagena et al. (2017). Ng (2006) interpreted the non-significant effect as a possible result of less demand on strategy use in the mathematics materials in junior high school than in senior high schools. Our results add another convincible interpretation by suggesting the existence of a certain reading proficiency threshold(s) that constrain the impact of reading strategies (i.e., somewhere near −0.71 standard units or 365 raw points).

The emphasis by Grimm (2008) and Ng et al. (2021) on the importance of linguistic resources provides thoughts for our discussion over the moderating effect of reading proficiency on strategy use. According to them, the first step for mathematical problem solving is to decode the print words to comprehend the problem and its given situation. Readers lacking these skills are likely to miscode information which in turn leads to miscomprehension or even confusion. Moreover, low reading proficiency is usually accompanied by low reading strategies (Oxford and Amerstorfer, 2018). Prompted by this miscoded information, students might activate inappropriate strategies, or although they activated the right strategy, their capacity in strategic processing prevented them from using it satisfactorily (Oxford and Amerstorfer, 2018).

Another possible reason explaining the inefficient use of reading strategies with struggling and low-proficiency readers might relate to their low capacity in working memory. A large volume of studies has shown that academic achievement is determined by individuals' working memory (Peng et al., 2018; Swanson, 2015) and students with low reading proficiency are usually also low in working memory capacity (Alloway, 2007; Fischbach et al., 2014; Kosmidis et al., 2011). The joining-up of low working memory capacity might have made struggling and low-proficiency readers' strategy use even worse.

The group of medium-proficiency readers contained 64 % of the total sample and benefited most from reading strategies on their mathematics achievement. This advantage over other groups should mostly result from their location nearing the middle point of the continuum of reading proficiency. When located near the middle, the beneficial potential of reading proficiency reaches its maximum (i.e., becoming saturated) and this saturation contributes to mathematics learning in at least two ways. First, the saturated potential directly flows to mathematical problem solving by facilitating the construction of the representation and situation models (Kintsch and Mangalath, 2011). Second, it removes the threshold that constrains the function of reading strategies and reading proficiency (Clarke, 1980a, 1980b).

When students' reading proficiency moved beyond a higher threshold (e.g., 1.29 standard units), mathematics achievement still received a positive effect from reading strategies but the effect size dropped. The stepping down of strategy effect is not only related to 'effect saturation' discussed earlier, it can also be related to readers' enhanced capacity in automatic reading processing and enhanced mathematical skills which eventually render reading strategies less useful (Cai and Kunnan, 2020). At this moment, higher linguistic resources and higher mathematical skills could now successfully build a mental representation toward relevant concepts embedded in the mathematical tasks at hand.

The drop can also be due to students' active adaption to reduce the cognitive load (Boekaerts, 2017; Seufert, 2018; Wirth et al., 2020) caused by the overuse of strategy to save energy for other activations. As demonstrated in previous studies (Fuchs et al., 2006; Ng et al., 2021), for higher-proficiency readers, their focus might be on the solution stage during which the students rely more on the activation of mathematical skills (e.g., algorithm skills, content knowledge) or other resources such as motivation (e.g., attention, motivation, confidence) to construct an appropriate solution.

To wrap up, the curvilinear relationship between reading strategies and mathematics was mostly due to the curvilinear relationship between reading strategies and the mediator reading proficiency. In line with the IRC theorems, three mechanisms might have determined this curvilinear relationship: the detrimental and beneficial potential of reading strategies (bipolarity), the maximum beneficial potential with medium-proficiency achievers (golden centrality or effect saturation), and self-adaptation of strategy use.

Our study also provided findings regarding the fluid effects of student variables (i.e., gender, SES, and home language) on reading and mathematics achievement. The literature prevails with the stereotype that girls are better at reading (Chiu and McBride-Chang, 2006; Logan and Johnston, 2009) and boys are better in mathematics (Baiduri et al., 2020; Liu and Wilson, 2009). Our study supported the gender stereotype regarding mathematics supported across all groups and this gender gap gradually enlarged with the increase in reading proficiency. However, the gender stereotype regarding reading was only partly supported by struggling readers and medium-proficiency readers. Combined, these results suggested that, although girls have an advantage in reading which facilitates mathematics learning, this advantage is limited in helping shrink the gender gap in mathematics.

The relation between SES and reading proficiency was found to be positive and largest with medium-proficiency readers, suggesting a possible IRC mechanism. The relation between SES and mathematics was positive and gradually attenuated with the increase in reading proficiency, an interesting finding rarely revealed in previous studies. The different pattern in which SES functions with reading and mathematics is interesting, however, the reason for this difference remains unknown.

The use of test language at home seemed to benefit reading proficiency and mathematics achievement across all reading proficiency groups, but the effect sizes were small. These results in general were consistent with existing studies about reading (Babayiğit and Shapiro, 2020; Raudszus et al., 2019) but not necessarily about mathematics. Studies set in high-income countries such as Ireland revealed using test language at home was related to lower mathematics scores (e.g., Gilleece et al., 2010), whereas the results turned oppositive in studies set in low-income countries such as South Africa (Delprato, 2021; Prinsloo and Harvey, 2020). It is perhaps the mixture of country income levels or cultures in the PISA data that has attenuated the relation between home language use and mathematics learning.

6. Conclusion, limitations, and implications

This study found that reading proficiency was positively related to mathematics achievement and reading strategies and the two relations fluctuated across different reading proficiency groups. Accordingly, reading strategies were indirectly related to mathematics achievement through reading proficiency and this indirect relation also fluctuated, with the largest effect in the medium-reading proficiency group. The pattern of these fluctuations is quite close to the IRC discovered with reading strategies in second language settings.

Our study bears several limitations. First, we only included domain-general reading strategies (i.e., understanding and memorizing strategies) that are available from PISA 2018. Future studies may consider including domain-specific reading strategies more directly related to mathematics learning such as proof-reading strategies (Weber, 2015).

Second, reading proficiency was grouped by subjective cut-off points. It is possible that grouping using different cut-off scores might produce different results. However, the additional analyses we conducted using the conventional scores of ± 1 standard units and the eight levels grouped by OECD (2020) also suggested the IRC pattern. Having said that, although our use of the subjective cutoff scores might not have not perfectly represented the reality, the main story remained stable and the results should be convincible for prompting useful thoughts for research and practice in reading strategies.

Third, given the targeted population provided by PISA 2018, the results are only generalizable to 15-year-olds in mathematics learning. Future studies may try to explore with students of other age spans and to extend the duration of observation to track the progression of reading strategies during mathematics learning.

Apart from focusing on reading strategies, future studies are also encouraged to explore the application of the IRC theory to other individual factors essential for student learning. Take for examples, cognition factors (e.g., self-regulation, executive functioning, critical thinking, systems thinking, design thinking, computational thinking, etc.) and cognition-based motivation factors (e.g., growth mindset, self-concept, self-efficacy, motivation regulation, emotion regulation, among others). when conducting this IRC-oriented enquiry, more attention should be paid to the efficiency of the activation as done with the usefulness of strategy use in the original IRC and PISA 2018. Finally, it is highly recommended that future studies take into account the perspective of the cognitive load theory when exploring the fluctuating effect of these learning factors.

Regardless of these limitations, our findings contribute to the literature in several ways. Theoretically, the identification of the fluctuating relation between reading strategies and mathematics learning verified our assumption for the necessity of applying the IRC to learning in domains other than language learning. More studies are needed to test this possibility in mathematics learning and other academic domains such as science, medicine, economics, and so forth. Besides, the IRC phenomenon might not only work with strategy use, but also with other cognition-demanding variables such as thinking skills (e.g., critical thinking, systems thinking, design thinking, computational thinking, problem-solving, etc.), or motivation variables that related to cognition (e.g., motivation or emotion regulation, self-concept, self-efficacy, growth mindset, etc.).

Practically, our results provide useful thoughts for strategy training in mathematics education. The variation of reading strategy effect across groups caveats the one-size-fits-all scheme for instruction on reading strategies. While instruction on reading strategies in mathematics classes is timely and appropriate for medium-reading proficiency readers, the intervention may not be equally appropriate for readers at proficiency levels.

For low-proficiency readers, while it may be helpful to teach them basic knowledge on reading strategies, it is too early to coach them intensively as they lack sufficient linguistic resources that allow for efficient activation of reading strategies. Besides, low-proficiency readers are usually also low in working memory capacity (Alloway, 2007), and the imposed cognitive load by ineffective strategy activation may render the situation worse. It is advised that before delivering intensive instruction on reading strategies more attention should be paid to enhance low-proficiency readers' linguistic knowledge.

For those high-proficiency readers, they have possessed adequate ability in text processing and they are usually competent strategy users. What they need more is training on domain-specific knowledge and mathematical problem-solving skills.

During the intervention, to maximize the effect of strategy instruction, students' reading proficiency and reading strategies may be diagnosed before the instruction begins. Besides, cross-disciplinary efforts between teachers in mathematics and language (either L1 or L2) should be encouraged to maximize the effect of instruction.

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Data availability statement

The data that support the findings of this study are openly available in OECD PISA 2018 dataset at: https://www.oecd.org/pisa/data/2018database/

Ethic approval

Not applicable.

Consent to participate

Not applicable.

Consent to publication

The author consents to publish this article in *Learning and Individual Differences*.

Declaration of competing interest

None.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.lindif.2022.102180.

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